ප්‍රතිලෝම ත්‍රිකෝණමිතික ශ්‍රිත [Inverse trigonometric functions]

38. පහත වීවා සාධනය කරන්න.

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ බව පෙන්වන්න.

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ බව පෙන්වන්න.

(iii) $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} = \tan^{-1}\frac{2}{9}$

(iv) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

(v) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1} = \frac{\pi}{4}$

(vi) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{3}{5}$

(vii) $\tan^{-1}\frac{1}{9} = \frac{\pi}{4} - \tan^{-1}\frac{4}{5}$

(viii) $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$

(ix) $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \tan^{-1}\frac{77}{85}$

(x) $\frac{\pi}{2} - \cos^{-1}\frac{3}{5} = \sin^{-1}\frac{3}{5}$

(xi) $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{16}{25} + \cos^{-1}\frac{12}{13} = \frac{\pi}{2}$

(xii) $\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{7}{17} = \frac{\pi}{4}$

(xiii) $\tan^{-1}x - \tan^{-1}\left(\frac{\sqrt{3}x-1}{\sqrt{3}x+1}\right) = \frac{\pi}{6}$

(xiv) $3\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{20} = \frac{\pi}{4} - \tan^{-1}\frac{1}{1985}$

(xv) $\tan^{-1}\frac{120}{119} = 2\sin^{-1}\frac{5}{13}$

(xvi) $\tan^{-1}\frac{a}{b} - \tan^{-1}\left(\frac{a-b}{a+b}\right) = \frac{\pi}{4}$

(xvii) $\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0$

(xviii) $\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{5}{13} = \cos^{-1}\left(\frac{-33}{65}\right)$

(xix) $\sin^{-1}A - \sin^{-1}B = \sin^{-1}[A\sqrt{1-B^2} - B\sqrt{1-A^2}]$

(xx) $\cos^{-1}A + \cos^{-1}B = \cos^{-1}[AB - \sqrt{(1-A^2)(1-B^2)}]$

39. පහත සමීකරණ විසඳන්න.

(i) $\tan^{-1} + \tan^{-1}2x = \frac{\pi}{4}$

(ii) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}2$

(iii) $\tan^{-1}\left(\frac{1}{y-1}\right) - \tan^{-1}\left(\frac{1}{y+1}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

(iv) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

(v) $\tan^{-1}x + \tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{2}$

(vi) $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$

(vii) $2\tan^{-1}\left(\frac{x}{2}\right) - \tan^{-1}\left(\frac{x}{7}\right) = \frac{\pi}{4}$ නම්, x ට ඇත්තේ එකම එක අගයක් බවද, එම අගය 1 බව ද සාධනය කරන්න.

(viii) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ නම් $x = \frac{1}{\sqrt{2}}$ හෝ $x = -2\sqrt{2}$ බව පෙන්වන්න.

(ix) $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$